Gas Turbine Performance
A Simple hand calculation method

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1-spool turbojet example

Performance analysis of a simple 1-spool Turbojet

```
   ________   ________   ________    ________   ________   ________   ________
  |        | |        | |        |  |        | |        | |        | |        |
  | INTAKE | | COMPRE | | PREMAS |  | BURNER | | MIXEES | | TURBIN | | NOZCON |
  |________| |________| |________|  |________| |________| |________| |________|
 1          2          3   |      4          5    |     6          7          8
                           |10                    |10
                           |______________________|
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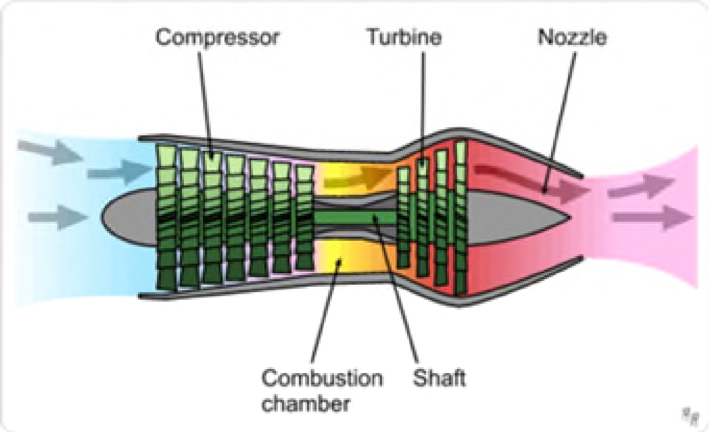
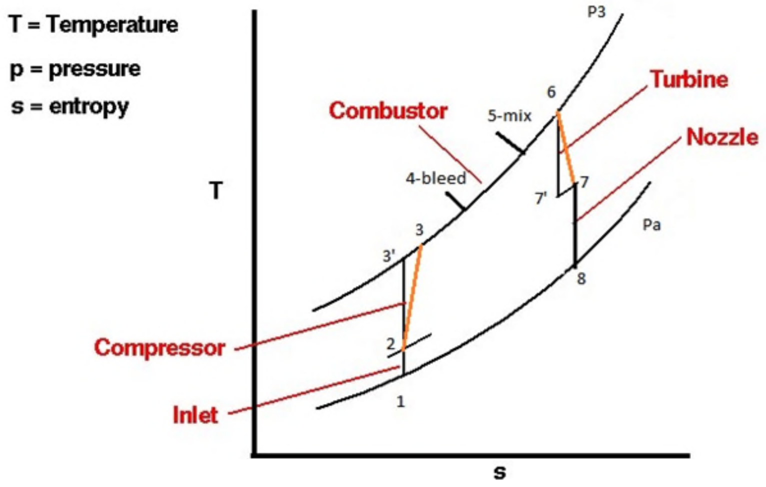
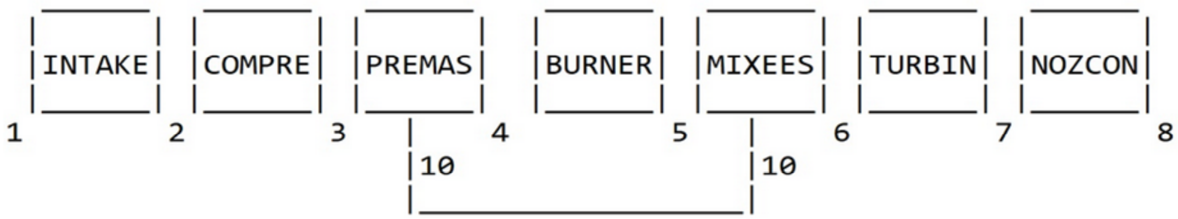


1 – Intake
2 – Compressor Inlet
3 – Compressor Exit
4 – Combustor Inlet
5 – Combustor Exit
6 – Turbine Inlet
7 - Turbine Exit
8 – Nozzle Exit



1-spool turbojet example – Design point hand calculations



Component Characteristics	Value
Altitude	0
Mach number	0
Compressor design pressure ratio	8.8
Compressor and Turbine efficiency	89%
Bleed mass flow	Nil
Combustor pressure loss	Nil
Inlet mass flow	77.2 kg/s
TET	1141K

Design point performance

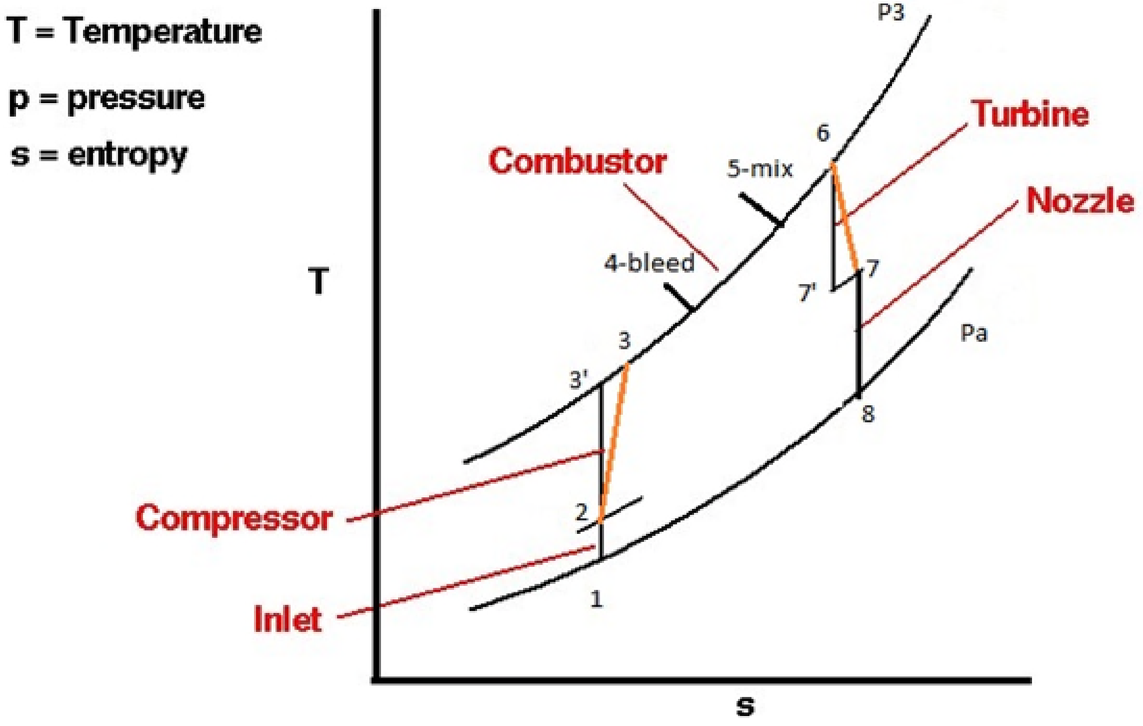
Compressor Calculations

- $\frac{P_3}{P_2} = (\frac{T_3'}{T_2})^{\frac{\gamma}{\gamma-1}}$
- T_3' = 288.15*(8.8)$^{\frac{1.4-1}{1.4}}$
- T_3' = 536.38K
- T_3-T_2= $\frac{T_3'-T_2}{\eta_C}$
- T_3 = 567.1K

Compressor Work = $W C_p \Delta T_{32}$

CW = 77.2*1005*(567.1-288.15)

CW = 21.64 MW



Design point performance

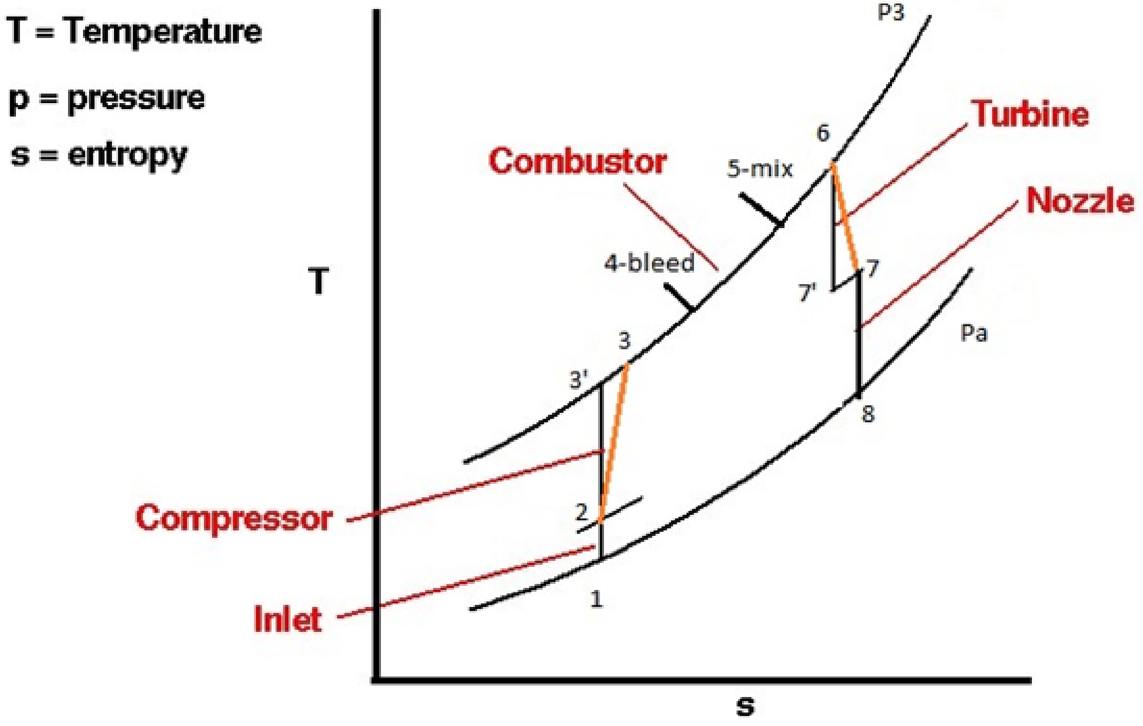
Combustor Calculations

- $P_3 = P_4 = 8.8$ bar
- $T_3 = T_4 = 567.1$K
- $Q_{in} = W C_p \Delta T_{45}$

  = 77.2*1150*(1141-567.1)

  = 50.96 MW
- $W_{ff} = Q_{in}$/LCV = 50.96/42.68

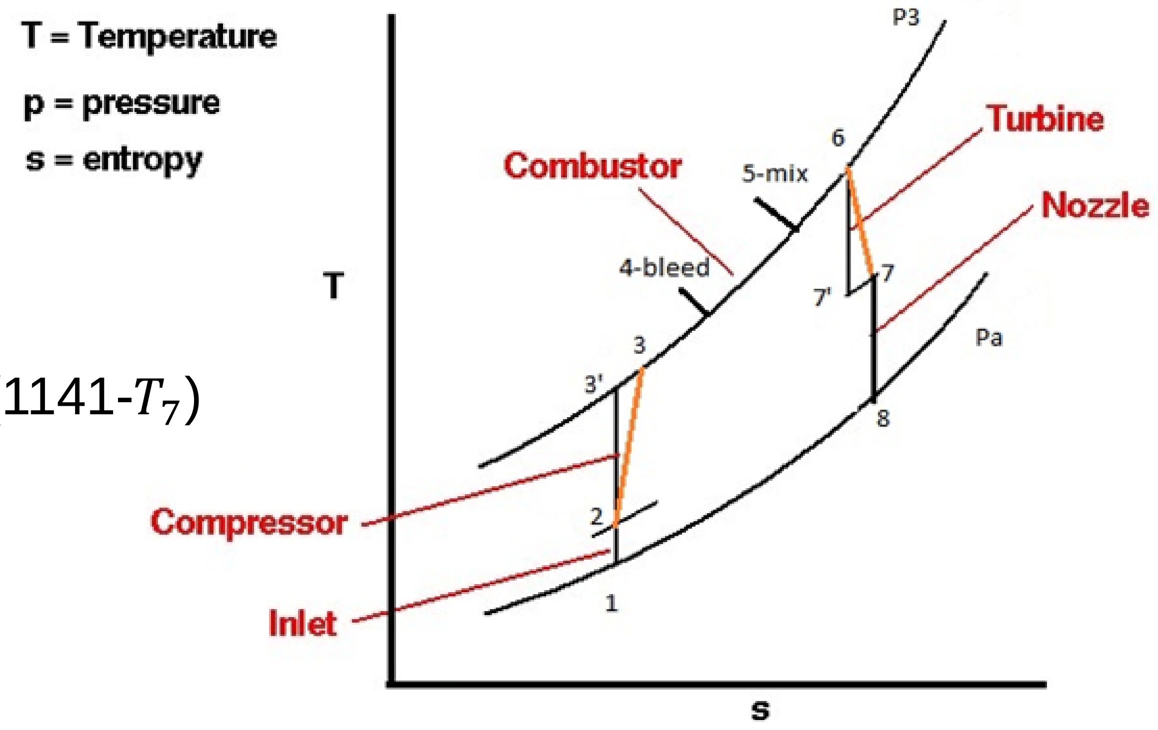
  = 1.194 kg/s



Design point performance

Turbine Calculations

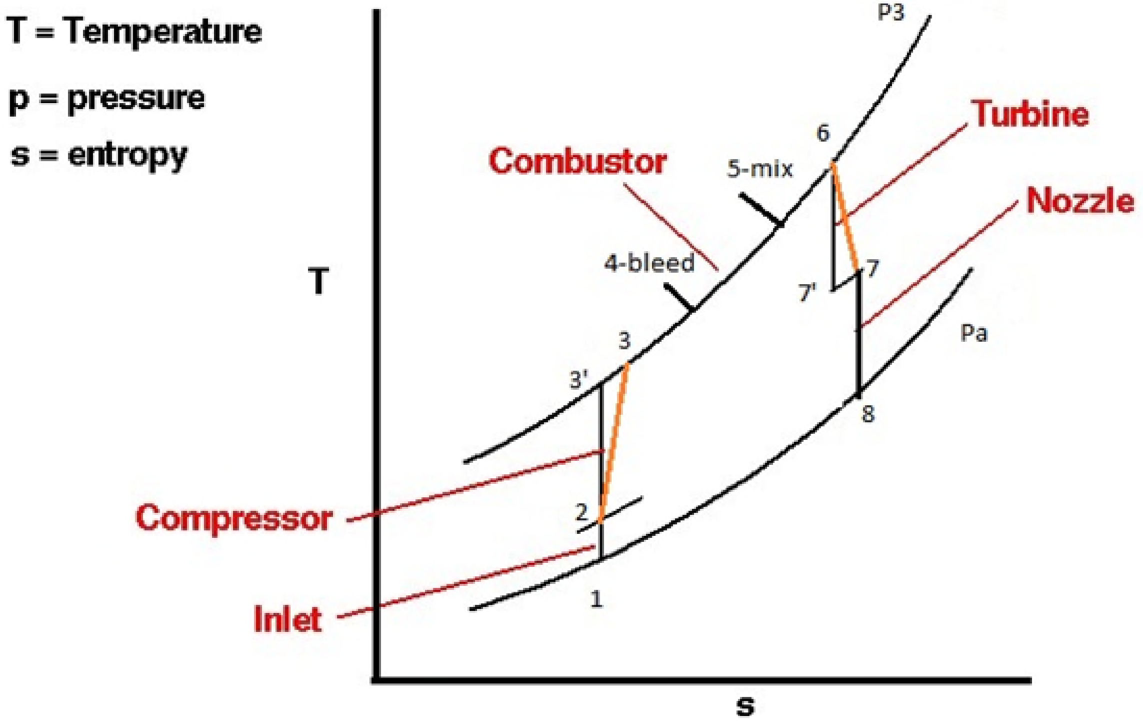
- $T_6 = T_5$ = 1141K
- W_{cold} = 77.2 kg/s
- W_{hot} = 78.394 kg/s
- CW = TW
- 21.64 x 10^6 = 78.394*1150*(1141-T_7)
- T_7 = 901K
- $\eta_t = \frac{T_6 - T_7}{T_6 - T_7'}$



Design point performance

Turbine Calculations

- $T_7{}' = 871.34\text{K}$
- $\gamma = 1.333$ (hot section)
- $\frac{P_6}{P_7} = \left(\frac{T_6}{T_7{}'}\right)^{\frac{\gamma}{\gamma-1}}$
- $\frac{P_6}{P_7} = 2.9427$
- $P_7 = 2.99 \text{ bar}$



Design point performance

Nozzle Calculations

Add the isentropic flow eq

- $\frac{P_7}{P_c} = 1.852; \frac{P_7}{P_a} = 2.99;$
- $\frac{P_7}{P_8} = 1.852 \left(As \frac{P_7}{P_c} < \frac{P_7}{P_8} \right) P_8$ = 1.614 bar (Choked nozzle)
- $\frac{T_7}{T_c}$ = 1.1665; T_8 (static) = 772.4K
- $V_8 = \sqrt{\gamma R T_8} = \sqrt{1.333 * 287 * 772.4}$ = 543.6 m/s
- $\rho_8 = \frac{P_8}{RT_8}$ = 0.7281 kg/m^3
- A = $\frac{W_8}{\rho_8 V_8}$ = 0.1981 m^2

Design point performance

Performance Calculations

- $F_N = W_{in}v_e + A_e(P_e - P_a)$
- F_N = 78.394*543.6 + 0.1981(1.614-1)*10^5
- F_N = 54.78 kN

Design performance – Simulation

Please use the WebEngine – 138.250.13.56 on your web browser

- Login using provided user ID
- Password is **isabe&2022**
- Use the **Run Engine** tab
- Please select **Turbojet_PCN** engine
- Scroll down and click **Set Operating Conditions**
- Click **Run Engine**
- Click **Results** and observe the output

- Note that there will be some discrepancy in calculations as the simulation uses variable **Cp and γ** values
- Hand calculations have used constant **Cp and γ** values

Off-Design performance – hand calculations

Off-design calculation steps

- Step – 1: To guess compressor pressure ratio
- Step – 2: To guess combustor temperature rise
- Step – 3: To guess turbine enthalpy drop
- Step – 4:

Determine compatibility of mass flow (W_{in} = W_{out})

Determine compatibility of rotation speed ($N_{compressor}$ = $N_{turbine}$)

Determine compatibility of work (CW = TW)

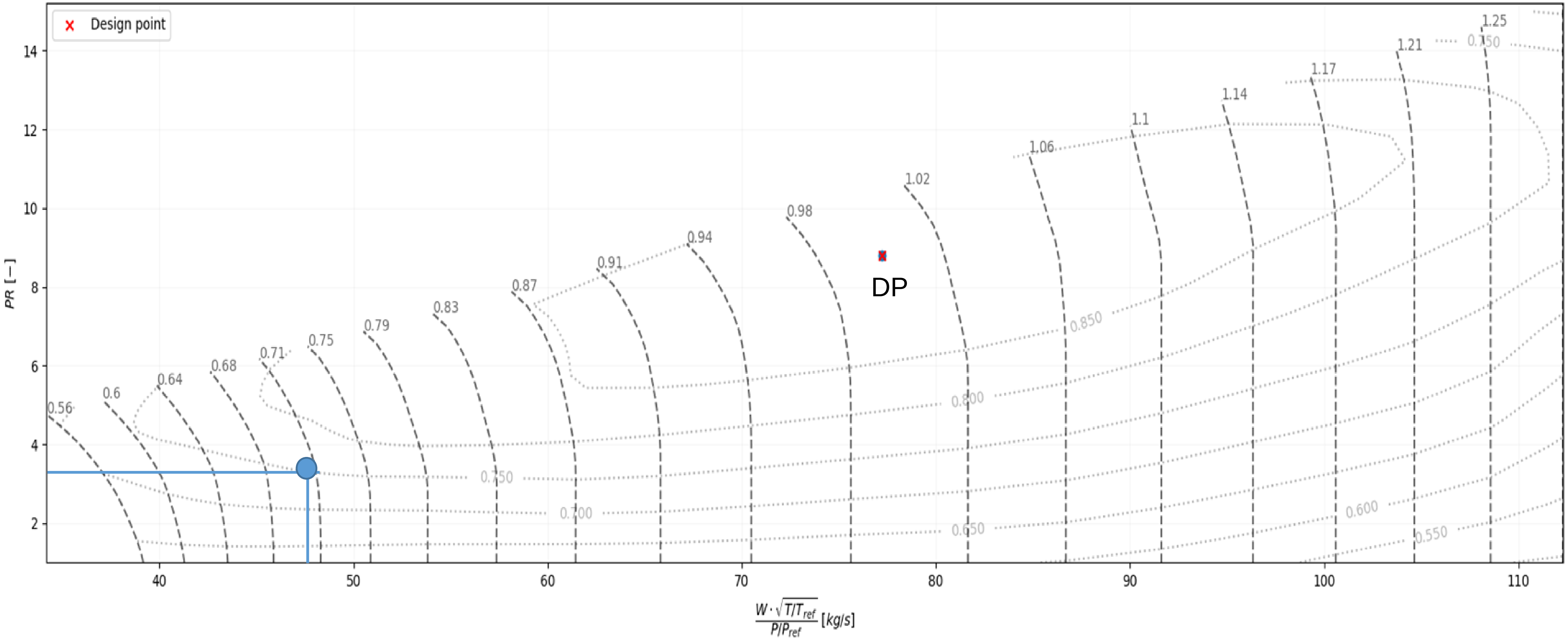
Off-Design performance – 1^{st} iteration

Step – 1

- Let us select an off-design handle for **PCN = 0.7** (70% rotation speed)
- Guess the compressor pressure ratio, $\frac{P_3}{P_2}$
- Obtain the Non-dimension mass flow and corresponding compressor efficiency from the compressor map.
- Compute the mass flow rate and temperature at compressor exit.

Off-Design performance – 1st iteration

Compressor characteristic map



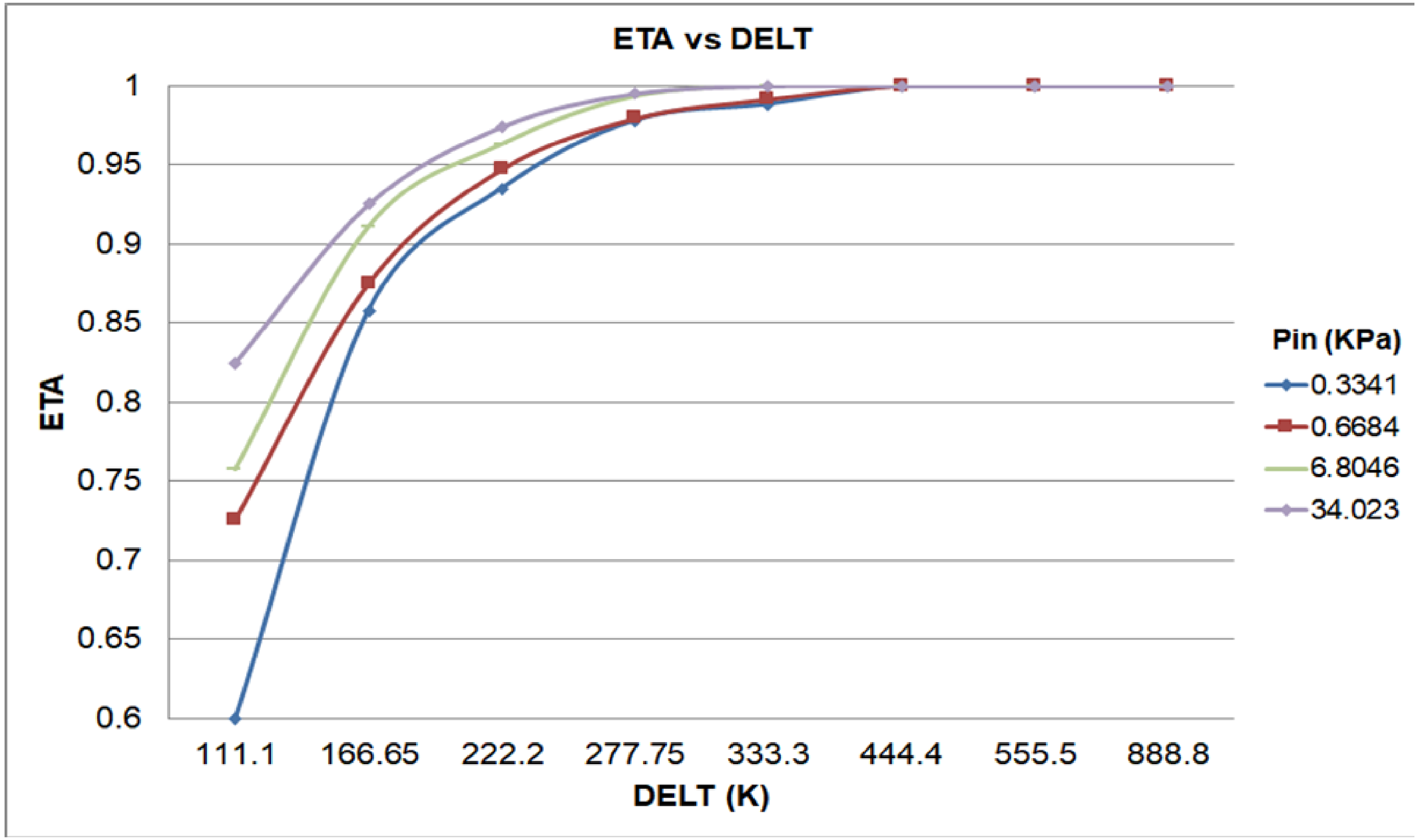
Off-Design performance – 1st iteration

Step – 1

- Guessed pressure ratio, $\frac{P_3}{P_2}$ = 3.3171
- Corresponding NDMF = $\frac{W_2\sqrt{T_2/T_{ISA}}}{P_2/P_{ISA}}$ = 48.094
- W_2= 48.094 kg/s
- Corresponding efficiency of compressor = 0.75
- T_3-T_2= $\frac{T_3'-T_2}{\eta_C}$; ΔT_{32} = 157K
- T_3 = 445.15K

Off-Design performance – 1st iteration

Combustor characteristic map



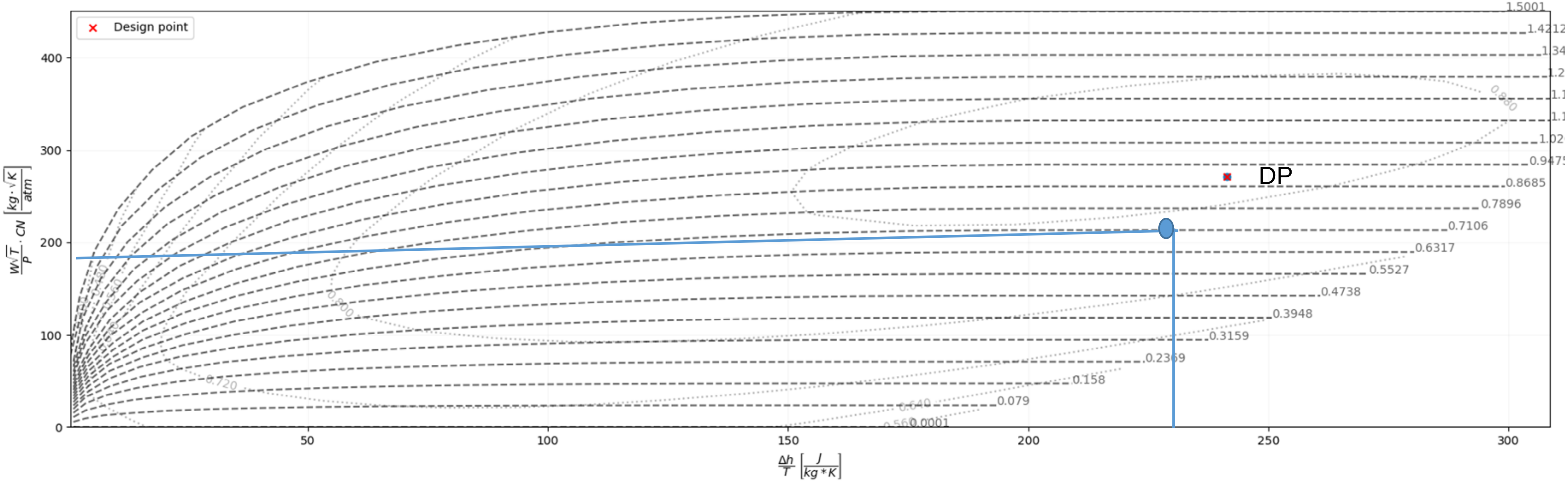
Off-Design performance – 1st iteration

Step-2

- Guess ΔT_{45} on the combustor map
- ΔT_{45} = 333K
- Corresponding η_{cc} = 1.0
- T_5 = 778.15K
- $Q_{in} = W C_p \Delta T_{45}$ = 48.094*1150*(778.15-445.15) = 18.42 MW
- $W_{ff} = Q_{in}$/LCV = 0.436 kg/s
- W_{hot} = 48.53 kg/s

Off-Design performance – 1st iteration

Turbine characteristic map



Off-Design performance – 1st iteration

Step-3

- Guess the turbine $\frac{\Delta H}{T}$
- $\frac{\Delta H}{T}$ = 236.67 J/kg-K
- T_5 = T_6 = 778.15K
- Corresponding NDMF * CN = 213.29
- Corresponding turbine efficiency = 0.86
- $\eta_t = \frac{T_6 - T_7}{T_6 - T_7{'}}$

Off-Design performance – 1st iteration

Step-3

- $\frac{\Delta H}{T_6} = \frac{C_p(T_6 - T_7)}{T_6}$ where C_p = 1150 kJ/kg-K; T_6 = 778.15K
- T_7 = 618K
- $\frac{T_6}{T_7{'}} = \frac{T_6 \eta_t}{T_6 \eta_t - (T_6 - T_7)} = \frac{778.15\ x\ 0.86}{778.15\ x\ 0.86 - (778.15 - 618)}$ = 1.3146
- $\frac{P_6}{P_7} = (\frac{T_6}{T_7{'}})^{\frac{\gamma}{\gamma - 1}}$ = 2.989 (γ = 1.333)
- P_7 = 1.11 bar

Off-Design performance – 1st iteration

Step-4

- Rotation Compatibility:

$$\frac{N}{\sqrt{T_5}} = \frac{N}{\sqrt{T_2}}\ x \frac{\sqrt{T_2}}{\sqrt{T_5}}$$

(N = 0.7; T_2= 288.15K; T_5 = 778.15K)

LHS = 0.02509

RHS = 0.02509

Note: *If the condition is not satisfied, an internal loop runs repeating Step-2 where Combustor Outlet Temperature is guessed again.*

Off-Design performance – 1st iteration

Step-4

- Mass Flow Compatibility:

$$\frac{W_5\sqrt{T_5/T_{ISA}}}{P_5/P_{ISA}} = \frac{W_2\sqrt{T_2/T_{ISA}}}{P_2/P_{ISA}}\, x\, \frac{P_2}{P_3}\, x\, \frac{P_3}{P_4}\, x\, \frac{\sqrt{T_5}}{\sqrt{T_2}}\, x\, \frac{W_5}{W_2}$$

(W_5= 48.53 kg/s; T_5 = 778.15K; P_5 = 3.3171 bar; W_2 = 48.094 kg/s; T_2 = 288.15K; P_2 = 1 bar)

LHS = 24.042

RHS = 24.042

Note: If the condition is not satisfied, an internal loop runs repeating Step-3 where turbine enthalpy drop is guessed again.

Off-Design performance – 1st iteration

Step-4

Is the Compressor Work = Turbine Work?

CW = 48.094*1005*(445.15-288.15)

CW = 7.589 MW

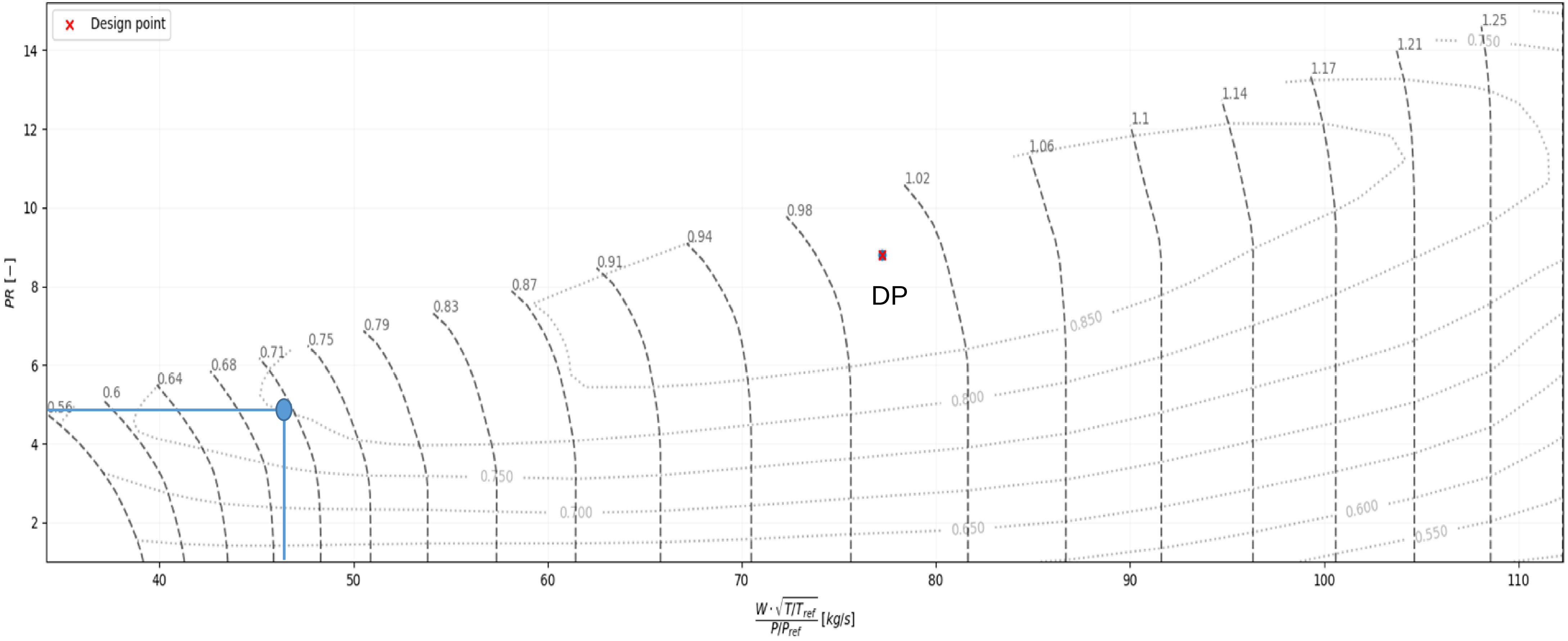
TW = 48.53*1150*(778.15-618)

TW = 8.938 MW

- The Compressor work is not equal to the Turbine work.
- Step – 1 is repeated where a new pressure ratio on 0.7 speed line is guessed.

Off-Design performance – 2nd iteration

Compressor characteristic map



Off-Design performance – 2nd iteration

Step – 1

- Guessed pressure ratio, $\frac{P_3}{P_2}$ = 4.514
- Corresponding NDMF = $\frac{W_2\sqrt{T_2/T_{ISA}}}{P_2/P_{ISA}}$ = 46.307
- W_2= 47.307 kg/s
- Corresponding efficiency of compressor = 0.791
- T_3-T_2= $\frac{T_3'-T_2}{\eta_C}$; ΔT_{32} = 194.31K
- T_3 = 482.46K

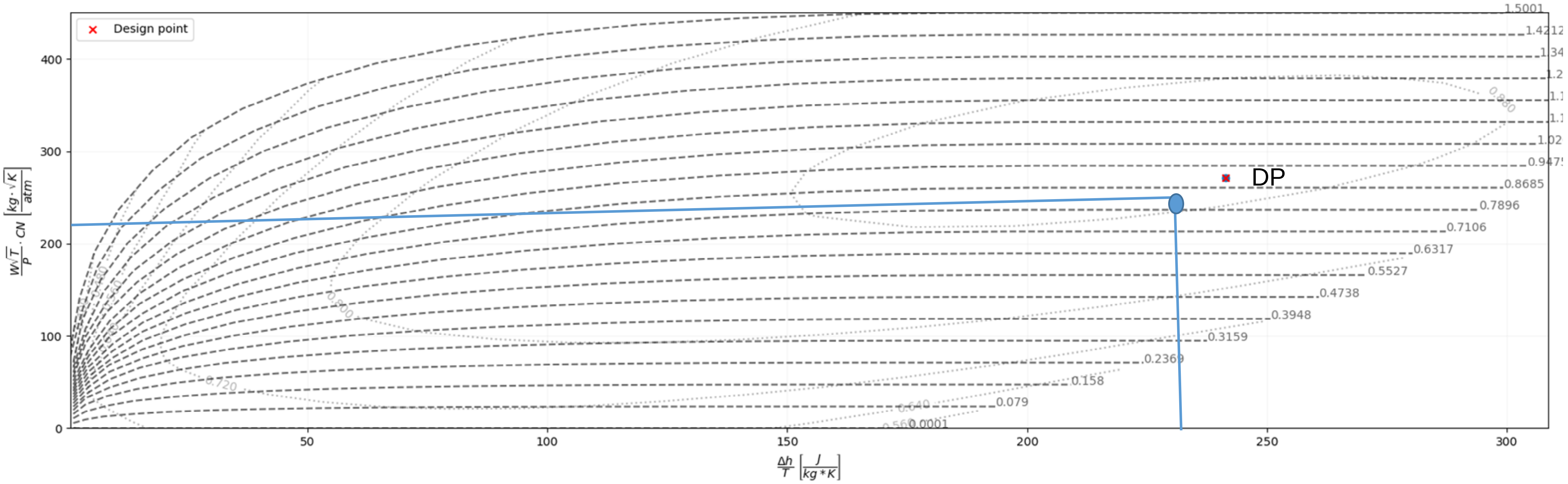
Off-Design performance – 2nd iteration

Step-2

- Guess ΔT_{45} on the combustor map
- ΔT_{45} = 359.34K
- Corresponding η_{cc} = 0.9996
- T_5 = 841.8K
- $Q_{in} = W C_p \Delta T_{45}$ = 46.307*1150*(841.8-482.46) = 19.136 MW
- $W_{ff} = Q_{in}$/LCV = 0.419 kg/s
- W_{hot} = 476.726 kg/s

Off-Design performance – 2nd iteration

Turbine characteristic map



Off-Design performance – 2nd iteration

Step-3

- Guess the turbine $\frac{\Delta H}{T}$
- $\frac{\Delta H}{T}$ = 244.234 J/kg-K
- T_5 = T_6 = 841.8K
- Corresponding NDMF * CN = 230.35
- Corresponding turbine efficiency = 0.872
- $\eta_t = \frac{T_6 - T_7}{T_6 - T_7{}'}$

Off-Design performance – 2nd iteration

Step-4

- Rotation Compatibility

$$\frac{N}{\sqrt{T_5}} = \frac{N}{\sqrt{T_2}}\ x\ \frac{\sqrt{T_2}}{\sqrt{T_5}}$$

(N = 0.7; T_2= 288.15K; T_5 = 841.8K)

LHS = 0.02413

RHS = 0.02413

Note: If the condition is not satisfied, an internal loop runs repeating Step-2 where Combustor Outlet Temperature is guessed again.

Off-Design performance – 2nd iteration

Step-4

- Mass Flow Compatibility

$$\frac{W_5\sqrt{T_5/T_{ISA}}}{P_5/P_{ISA}} = \frac{W_2\sqrt{T_2/T_{ISA}}}{P_2/P_{ISA}} \, x \, \frac{P_2}{P_3} \, x \, \frac{P_3}{P_4} \, x \, \frac{\sqrt{T_5}}{\sqrt{T_2}} \, x \, \frac{W_5}{W_2}$$

(W_5= 46.726 kg/s; T_5 = 841.8K; P_5 = 4.514 bar; W_2 = 46.307 kg/s; T_2 = 288.15K; P_2 = 1 bar)

LHS = 17.6926

RHS = 17.6926

Note: If the condition is not satisfied, an internal loop runs repeating Step-3 where turbine enthalpy drop is guessed again.

Off-Design performance – 2nd iteration

Step-4

- Work Compatibility

Is the Compressor Work = Turbine Work?

CW = 9.113 MW

TW = 9.113 MW

- The Compressor work is equal to the Turbine work. And the solution is now converged.
- Engine performance parameters can now be calculated
- The next step is to follow the calculation to the nozzle from the turbine exit to check the choking condition

Off-Design performance – 2nd iteration

Nozzle Calculations

- $\frac{P_7}{P_c}$ = 1.852; $\frac{P_7}{P_8}$ = 1.564
- P_8 = 1 bar as nozzle is unchoked ($\frac{P_7}{P_c}$>$\frac{P_7}{P_8}$)
- T_8 = 663.02K; t_8 = 586.4K
- $T_8 = t_8 + v_8^2/2C_p$
- v_8 = 402.9 m/s
- F_N = $W_{in}v_e$ = 18.55kN

Off-Design performance – Simulation

Please use the WebEngine – 138.250.13.56 on your web browser

- Use the **Run Engine** tab
- Please select **Turbojet_PCN** engine
- Select **PCN** as **Power Setting** and provide a value of **0.7**
- Scroll down and click **Set Operating Conditions**
- Click **Run Engine**
- Click **Results** and observe the output

- Note that there will be some discrepancy in calculations as the simulation uses variable **Cp and** γ values
- Hand calculations have used constant **Cp and** γ values

Off-Design performance – Conclusions

- The engine simulation is an iterative process to calculate off-design performance.
- The accuracy of the solution depends on the level of detail and accuracy of component maps available for performing the calculations.
- The engine simulation did not involve bleeds, combustor pressure loss, flow mixing. The level of complexity increases with double and triple spool as compatibility has to be performed on all shafts.
- The alternate method for the obtaining solution is – Matrix method

Off-Design performance – Simulation exercise 1

Please use the WebEngine – 138.250.13.56 on your web browser

- Use the **Run Engine** tab
- Please select **Turbojet_TET** engine
- Please complete the table below

SL	Operating condition	Compressor				Turbine				Performance			
		Pressure ratio	Exit temperature	Efficiency	Work	Pressure ratio	Exit temperature	Efficiency	Work	Net thrust	SFC	Mass flow	Fuel flow
1	Design point TET = 1141K												
2	OP1 TET = 900K												
3	OP2 TET = 1000K												
4	OP3 TET = 1200K												
5	OP4 TET = 1300K												

Off-Design performance – Simulation exercise 2

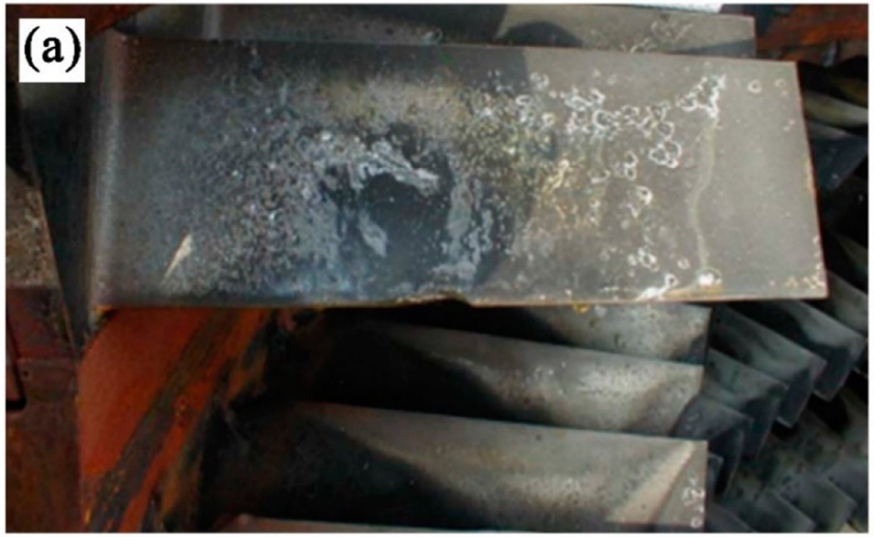
- Please select **Turbojet_TET** engine
- Please complete the table below (TET is constant = 1141K)
- Operating condition – (Altitude, Temperature deviation, Mach no.)

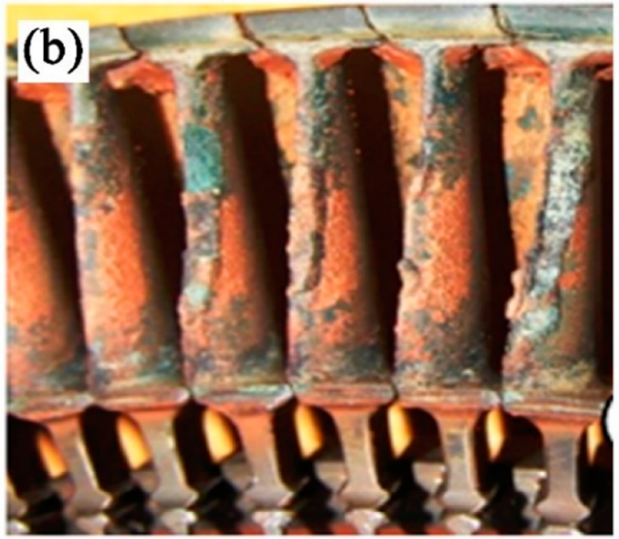
SL	Operating condition (Altitude, Temp dev, Mach no)	Compressor				Turbine				Performance			
		Pressure ratio	Exit temperature	Efficiency	Work	Pressure ratio	Exit temperature	Efficiency	Work	Net thrust	SFC	Mass flow	Fuel flow
1	Design point 0, 0, 0												
2	OP1 0, -15, 0												
3	OP2 2000, -10, 0.5												
4	OP3 5000, 0, 0.6												
5	OP4 8000, 10, 0.7												
6	OP5 10000, 0, 0.8												
7	OP6 6000, 20, 0.8												
8	OP7 0, 0, 0.6												

Off-Design performance – Degradation

Component degradation

- Engine components undergo degradation through its lifetime
- Common faults – fouling, erosion, shroud clearance
- Causes reduction in engine output



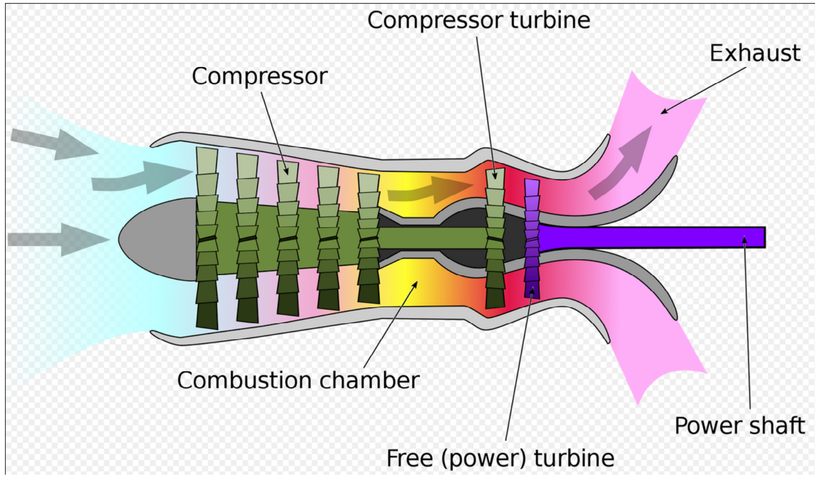


Turboshaft example

Performance analysis of helicopter turboshaft engine

```
 _______   _______   _______     _______   _______   _______
|       | |       | |       |   |       | |       | |       |
|INTAKE | |COMPRE | |BURNER |   |TURBIN | |TURBIN | |NOZCON |
|_______| |_______| |_______|   |_______| |_______| |_______|
1         2         3         4           5         6         7
```

1 – Intake
2 – Compressor
3 – Combustor
4 – Compressor turbine
5 – Power turbine
6 – Exhaust



Degradation – Simulation exercise 3

Please use the WebEngine – 138.250.13.56 on your web browser

- Use the **Run Engine** tab
- Please select **Turboshaft** engine

	Compressor faults			Turbine faults		
Operating condition	PR Deg	FC Deg	ETA Deg	DH deg	TF deg	ETA deg
OP1	0%	-1%	-2%	0%	0%	0%
OP2	-2%	0%	0%	0%	0%	-2%
OP3	0%	0%	0%	-1%	0%	-3%
OP4	-1%	-1%	-3%	0%	0%	0%
OP5	0%	0%	-4%	0%	0%	-2%
OP6	0%	0%	0%	-2%	-3%	0%
OP7	-3%	2%	-3%	0%	-1%	-1%
OP8	0%	1%	0%	0%	-2%	-4%

Degradation – Simulation exercise 3

Please use the WebEngine – 138.250.13.56 on your web browser

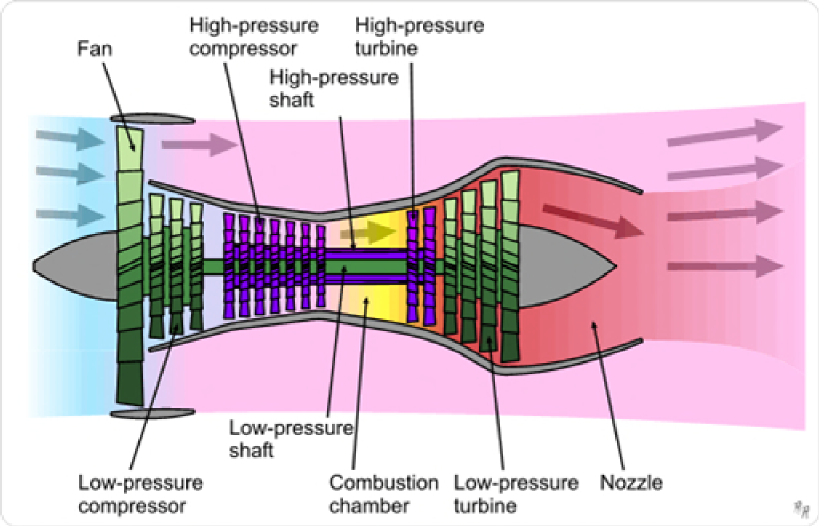
- Use the **Run Engine** tab
- Please select **Turboshaft** engine and impose the degradation conditions

SL	Operating condition	Compressor				Turbine				Power Turbine				Performance			
		Pressure ratio	Exit temperature	Efficiency	Work	Pressure ratio	Exit temperature	Efficiency	Work	Pressure ratio	Exit temperature	Efficiency	Work	Power	SFC	Mass flow	Fuel flow
1	Design point																
2	OP1																
3	OP2																
4	OP3																
5	OP5																
6	OP6																
7	OP7																
8	OP8																

2-spool turbofan example

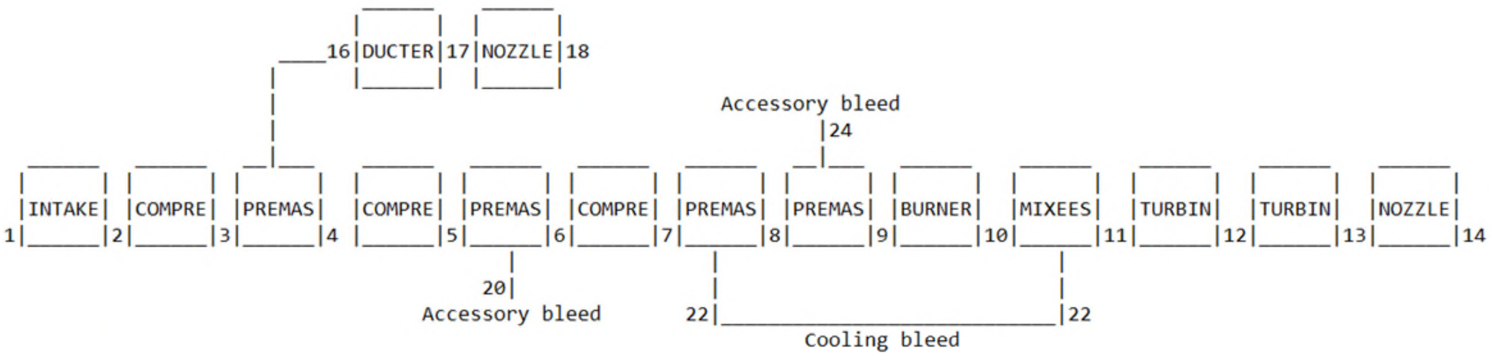
Performance analysis of 2-spool turbofan engine

```
                          ______    ______
                         |      |  |      |
                    ____16|DUCTER|17|NOZZLE|18
                   |     |______|  |______|
                   |
                   |                              Accessory bleed
                   |                                     |24
  ______   ______  __|___   ______   ______   ______   ______   __|___   ______    ______    ______    ______    ______
 |      | |      | |      |  |      | |      | |      | |      | |      | |      |  |      |  |      |  |      |  |      |
 |INTAKE| |COMPRE| |PREMAS|  |COMPRE| |PREMAS| |COMPRE| |PREMAS| |PREMAS| |BURNER|  |MIXEES|  |TURBIN|  |TURBIN|  |NOZZLE|
1|______|2|______|3|______|4 |______|5|______|6|______|7|______|8|______|9|______|10|______|11|______|12|______|13|______|14
                                      |                  |                                |
                                    20|                  |                                |
                              Accessory bleed          22|________________________________|22
                                                                  Cooling bleed
```



1 – Intake
2 – Fan
3 – Bypass flow
4 – LP compressor
5 – LP Bleed
6 – HP compressor
7 – HP bleed 1
8 – HP bleed 2
9 – Combustor
10 – Core and SAS mix
11 – HP turbine
12 – LP turbine
13 – Core nozzle
17 – Bypass nozzle
22 – Turbine cooling bleed
20 – LP accessory bleed
24 – HP accessory bleed

2-spool turbofan example – Design point hand calculations



Component Characteristics	Value
Altitude	10668m
Mach number	0.8
Fan pressure ratio	1.65
Fan efficiency	90%
Bypass ratio	5.5
Bypass pressure loss	1%
Inlet mass flow	170 kg/s
TET	1360K

Component Characteristics	Value
LP compressor pressure ratio	2.36
HP compressor pressure ratio	8.45
LP and HP compressor efficiency	88%
LP accessory bleed	Nil
HP cooling bleed	13%
HP accessory bleed	1%
Combustor pressure loss	5%
Combustion efficiency	99.9%

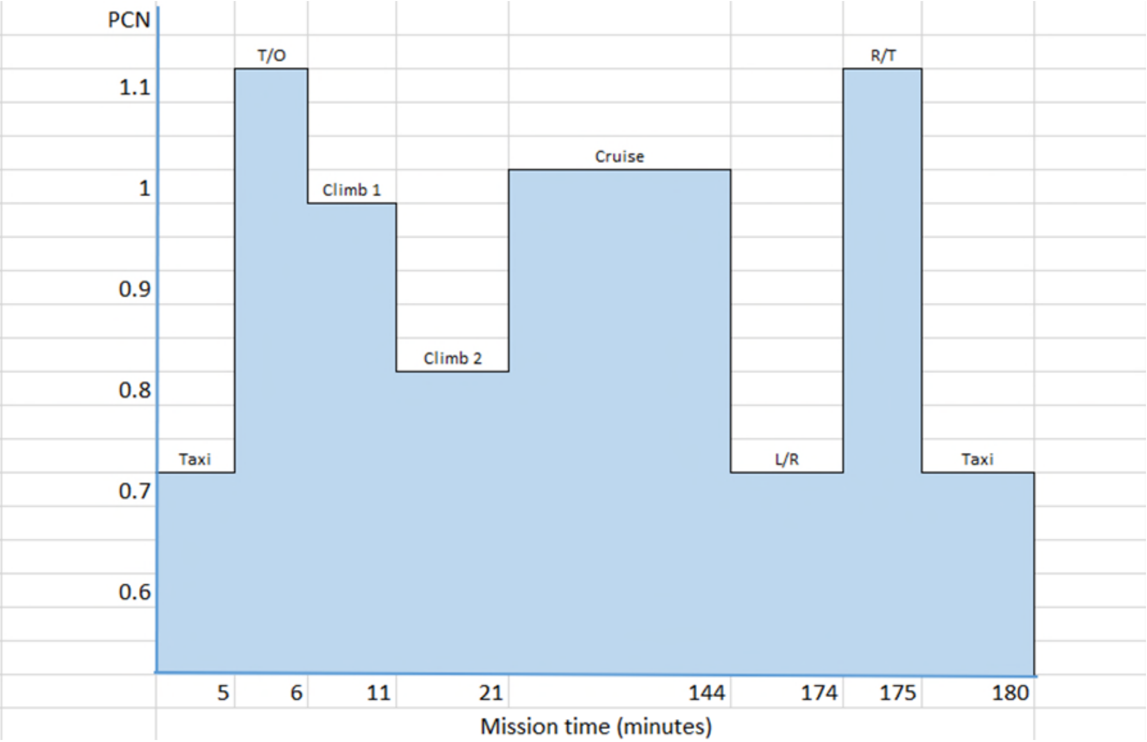
Component Characteristics	Value
HP turbine auxiliary work	500 W
HP compressor efficiency	90%
LP compressor auxiliary work	500 W
LP compressor efficiency	93%
Engine handle	TET/PCN
DP net thrust	25.7 kN
DP fuel flow	0.4 kg/s

Off-Design performance – Simulation exercise 4

Please use the WebEngine – 138.250.13.56 on your web browser

- Use the **Run Engine** tab
- Please select **Turbofan_PCN** engine

Operating condition	Time (mins)	Input		
		Altitude (m)	Mach number	PCN
Taxi	5	0	0	0.7
Takeoff	1	0	0	1.1
Climb 1	5	3000	0.5	0.9
Climb 2	10	7000	0.7	0.8
Cruise	123	10000	0.8	1
Low Rating	30	5000	0.75	0.7
Reverse Thrust	1	0	0.4	1.1
Taxi	5	0	0	0.7



Off-Design performance – Simulation exercise 4

- The temperature-stress and stress-LMP relations are provided below:

TET (K)	Stress (MPa)
1570	350
1525	300
1290	265
1215	250
1070	200
1040	150
900	100

Stress (MPa)	P
100	37.0
150	36.0
200	35.0
250	34.5
265	34.1
300	34.0
350	33.9

Assuming that we have been able to determine the operating temperatures and stress levels, from the Larson-Miller chart we can provide the parameter P in the equation:

- $P = \frac{T}{1000}(Log\ t_f + 20)$

- Where,
  T = Temperature K (TET)
  t_f = Time to failure

Off-Design performance – Simulation exercise 4

- Calculate the number of cycles the engine could operate in this mode until failure. Also apply a factor of safety of 1.5 and compute the hours of operation the engine can sustain based on LMP.

Operating condition	Time (mins)	Input			Output			Stress (Mpa)	P	t_f (hours)
		Altitude (m)	Mach number	PCN	Net Thrust	Fuel flow	Turbine inlet T11			
Taxi	5	0	0	0.7						
Takeoff	1	0	0	1.1						
Climb 1	5	3000	0.5	0.9						
Climb 2	10	7000	0.7	0.8						
Cruise	123	10000	0.8	1						
Low Rating	30	5000	0.75	0.7						
Reverse Thrust	1	0	0.4	1.1						
Taxi	5	0	0	0.7						

Off-Design performance – Simulation exercise 4

Operating condition	Time (mins)	t_f (hours)	T/t_f
Taxi	5		
Takeoff	1		
Climb 1	5		
Climb 2	10		
Cruise	123		
Low Rating	30		
Reverse Thrust	1		
Taxi	5		
Total	180		

- Total number of cycles using Miner's Law, $N = 1/(\text{Total } T/t_f)$
- Applying FOS = 1.5, the number of cycles, Nfos = N/1.5
- Total number of hours before failure criteria, T = Nfos x 180/60

Off-Design performance – Simulation exercise 5

- Consider Fan efficiency degradation of 3%. Estimate the reduction in the operating hours due to the faulty component for the same mission profile

- Enter -3 in Compressor efficiency degradation index (Corresponds to 97% fan efficiency scaling)

		Input			Output					
Operating condition	Time (mins)	Altitude (m)	Mach number	PCN	Net Thrust	Fuel flow	Turbine inlet T11	Stress (Mpa)	P	t_f (hours)
Taxi	5	0	0	0.7						
Takeoff	1	0	0	1.1						
Climb 1	5	3000	0.5	0.9						
Climb 2	10	7000	0.7	0.8						
Cruise	123	10000	0.8	1						
Low Rating	30	5000	0.75	0.7						
Reverse Thrust	1	0	0.4	1.1						
Taxi	5	0	0	0.7						

Off-Design performance – Simulation exercise 5

Operating condition	Time (mins)	t_f (hours)	T/t_f
Taxi	5		
Takeoff	1		
Climb 1	5		
Climb 2	10		
Cruise	123		
Low Rating	30		
Reverse Thrust	1		
Taxi	5		
Total	180		

- ❑ Total number of cycles using Miner's Law, $N = 1/(\text{Total } T/t_f)$
- ❑ Applying FOS = 1.5, the number of cycles, Nfos = N/1.5
- ❑ Total number of hours before failure criteria, T = Nfos x 180/60

Thank You

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